Introduction to Calibration Theory

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Computational Economics

- Econometrics and Programming approaches
  - Historically these approaches have been at odds, but recent advances have started to close this gap
- Advantages of Programming over Econometrics
  - Ability to use minimal data sets
  - Ability to calibrate on a disaggregated basis
  - Ability to interact with and include information from engineering and bio-physical models,
- Where do we apply programming models?
  - Explain observed outcomes
  - Predict economic phenomena
  - Influence economic outcomes

Economic Models

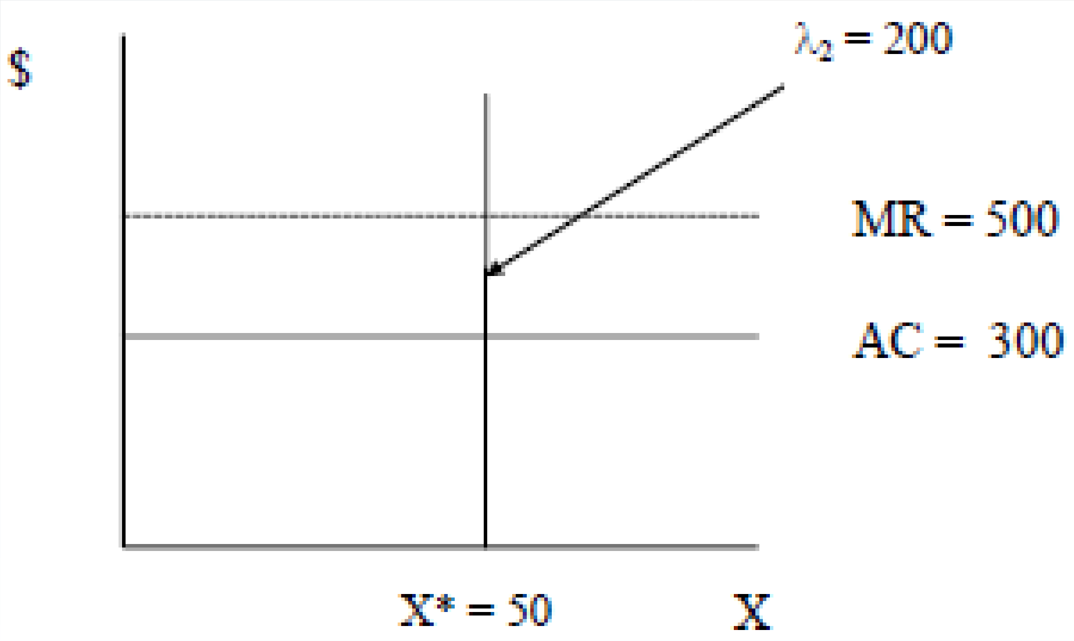
- Econometric Models
  - Often more flexible and theoretically consistent, however not often used with disaggregated empirical microeconomic policy models of agricultural production
- Constrained Structural Optimization (Programming)
  - Ability to reproduce detailed constrained output decisions with minimal data requirements, at the cost of restrictive (and often unrealistic) constraints
- Positive Mathematical Programming (PMP)
  - Uses the observed allocations of crops and livestock to derive nonlinear cost functions that calibrate the model without adding unrealistic constraints

Positive Mathematical Programming

- Behavioral Calibration Theory
  - We need our calibrated model to reproduce observed outcomes without imposing restrictive calibration constraints
- Nonlinear Calibration Proposition
  - Objective function must be nonlinear in at least some of the activities
- Calibration Dimension Proposition
  - Ability to calibrate the model with complete accuracy depends on the number of nonlinear terms that can be independently calibrated

PMP Cost-Based Calibration

- Let marginal revenue = $500/acre
- Average cost = $300/acre
- Observed acreage allocation = 50 acres



PMP Cost-Based Calibration

- Define a quadratic total cost function:

$$TC = \alpha x + 0.5\gamma x^2$$

$$MC = \alpha + \gamma x$$

$$AC = \alpha + 0.5\gamma x$$

- Optimization requires: MR=MC at x=50

- We can calculate γ *and* α sequentially,

$$\lambda_2 = MC - AC$$

$$\lambda_2 = MC - AC = 0.5\gamma x$$

$$\gamma = \frac{2\lambda}{x^*} = 8 \quad and \quad 300 = \alpha + 0.5 * 8 * 50$$

PMP Cost-Based Calibration

- We can then combine this information into the unconstrained (calibrated) quadratic cost problem:

$$\max \Pi = 500x - \alpha x - 0.5\gamma x^2 = 500x - 100x - 4x^2$$

- Standard optimization shows that the model calibrates when:

$$\frac{\partial \Pi}{\partial x} = 0 \Rightarrow x^* = 50$$

PMP With Multiple Crops

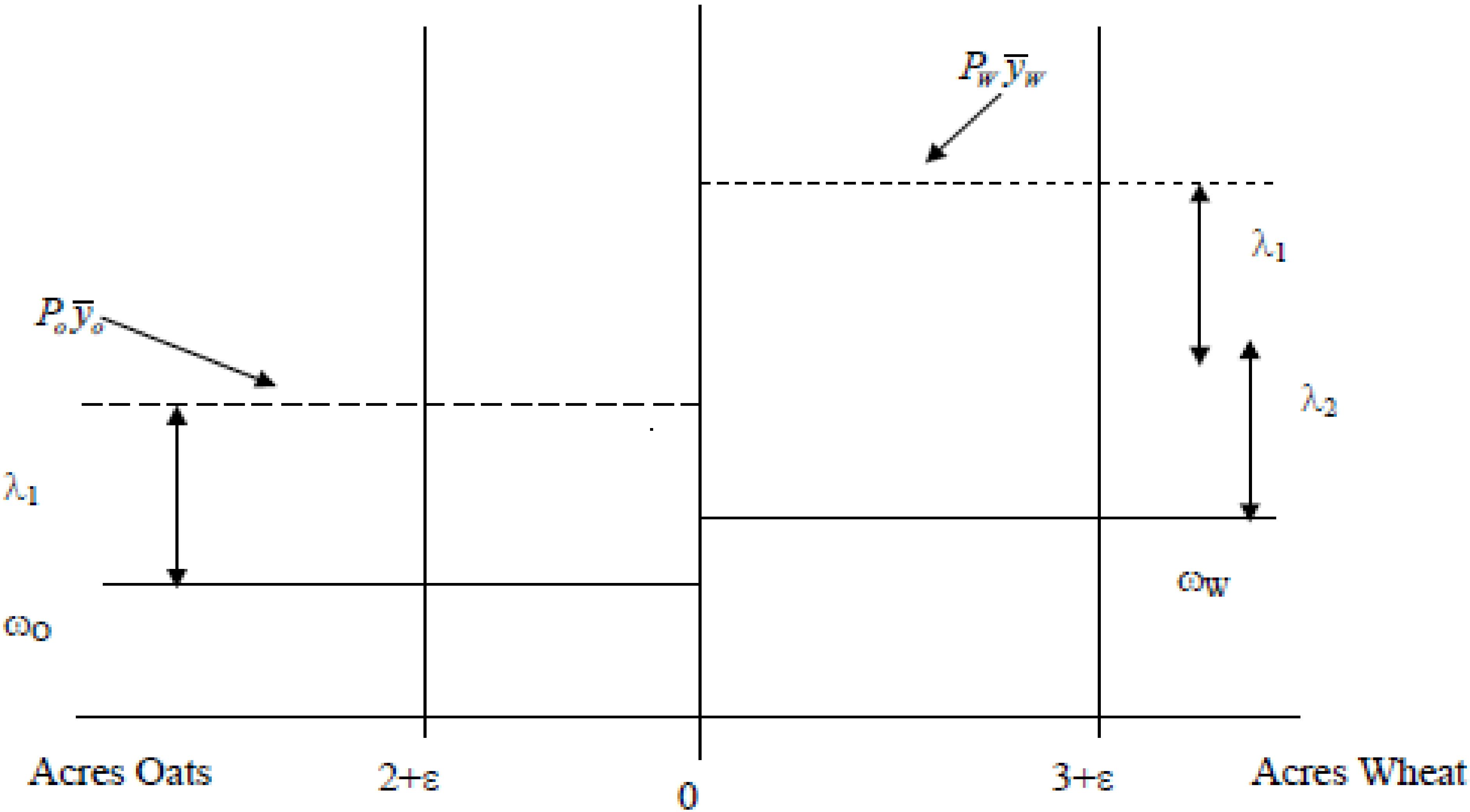
- Empirical Calibration Model Overview
  - Three stages:
    1) Constrained LP model is used to derive the dual values for both the resource and calibration constraints, $\lambda 1$ and $\lambda 2$ respectively.
    2) The calibrating constraint dual values ($\lambda 2$) are used, along with the data based average yield function, to uniquely derive the calibrating cost function parameters (α_i)and (γ_i).
    3) The cost parameters are used with the base year data to specify the PMP model.

PMP Example

- 2 Crops: Wheat and Oats
- Observe: 3 acres of wheat and 2 acres of oats

	Wheat (w)	(Oats) (o)
Crop prices	Pw = \$2.98/bu.	Po = \$2.20/bu.
Variable cost/acre	ww = \$129.62	wo = \$109.98
Average yield/acre	w = 69 bu.	o = 65.9 bu.

PMP Graphical Example



PMP Example (Stage 1)

- We can write the LP problem as:

$$\max \Pi = (2.98*69-130)x_w + (2.20*65.9-110)x_o$$

$$subject\,to$$

$$x_w + x_o \leq 5$$

$$x_w \leq 3+\varepsilon$$

$$x_o \leq 2+\varepsilon$$

- Note the addition of a perturbation term to decouple resource and calibration constraints

PMP Example (Stage 2)

- We again assume a quadratic total land cost function and now solve for $\alpha_i \; and \; \gamma_i$
- First: $\nabla f(\tilde{x}_k) = \lambda_{2k} \; ; \; 0.5\gamma_k \tilde{x}_k = \lambda_{2k} \; ; \; \gamma_k = \dfrac{2\lambda_{2k}}{\tilde{x}_k}$
- Second: $\sum_{ij} w_{ij} a_{ij} = c_i = \alpha_i + 0.5\gamma_i x_i$
- Therefore: $\alpha_i = c_i - 0.5\gamma_i x_i$

PMP Example (Stage 3)

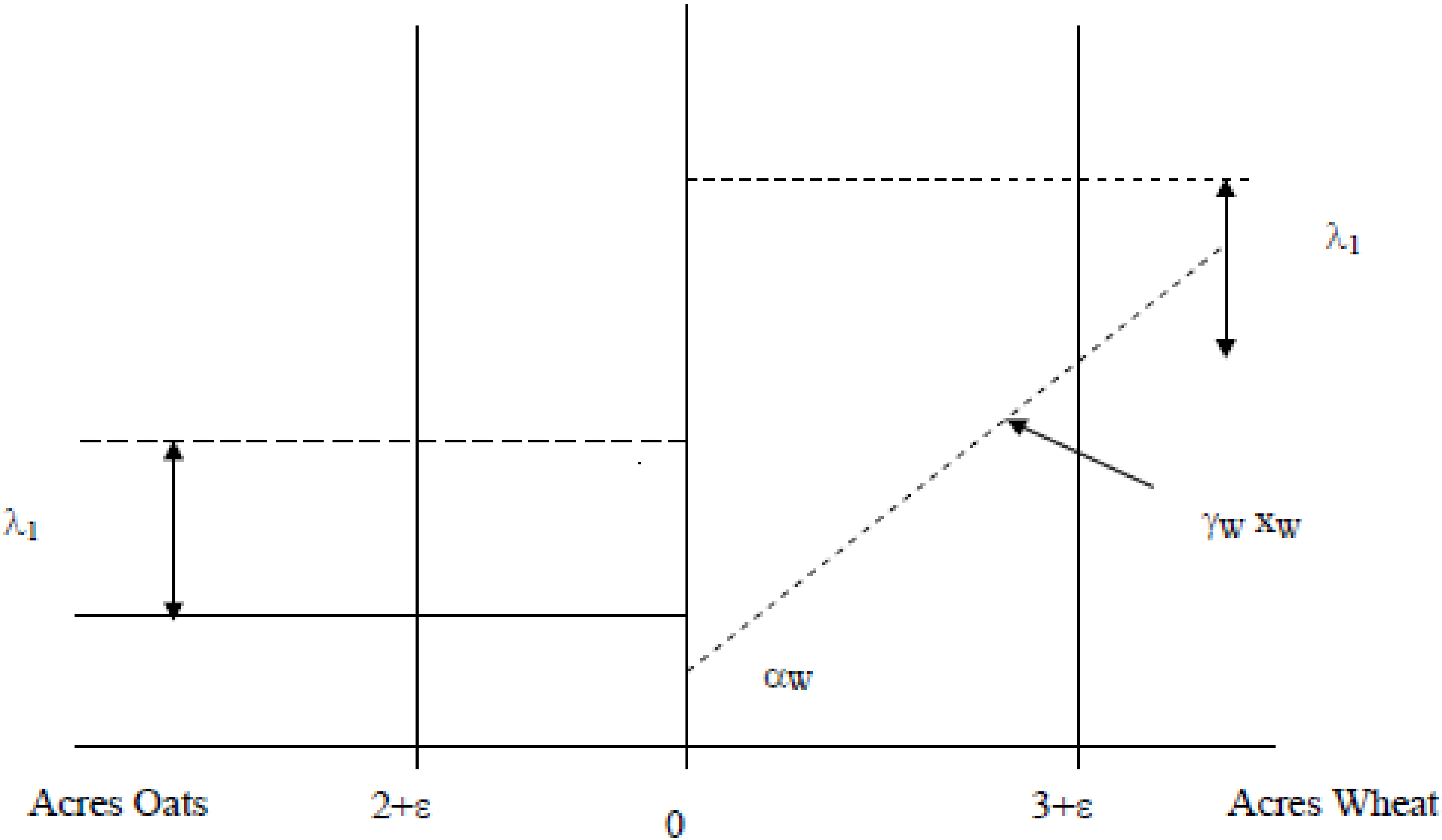
- After some algebra we can write the calibrated problem as and verify calibration in VMP and acreage:

$$\max \Pi = (2.98*69)x_w + (2.20*65.9)x_o - (88.62 + 0.5*27.33x_w)x_w - 109.98x_o$$

$$\textit{subject to}$$

$$x_w + x_o \leq 5$$

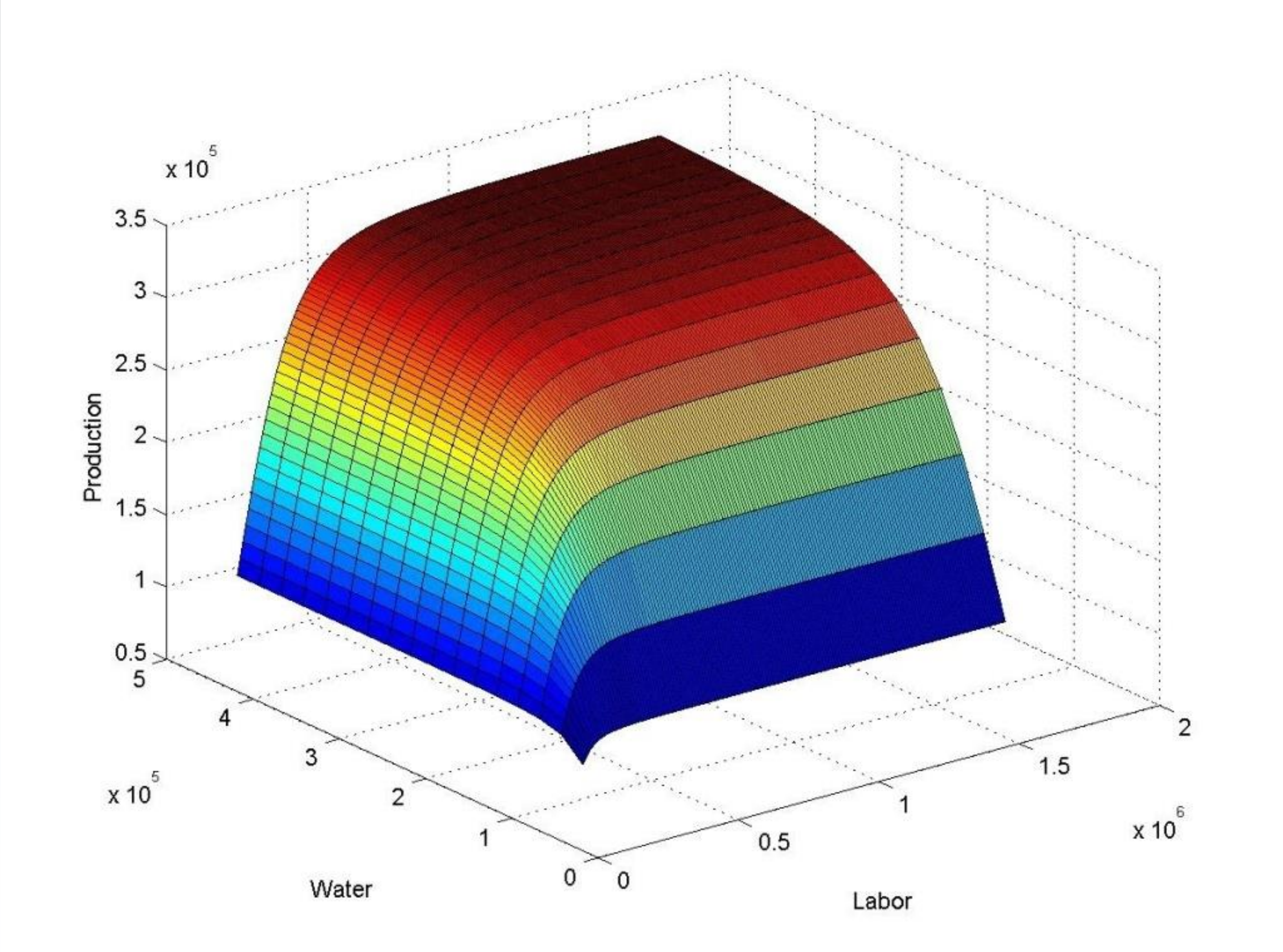
PMP Graphical Example



SWAP PMP Calibration

- We will consider a multi-region and multi-crop model where base production may be constrained by water or land
- CES Production Function
  - Constant Elasticity of Substitution (CES) productions allow for limited substitutability between inputs
- Exponential Land Cost Function
  - We will use an exponential instead of quadratic total cost function

CES Production Function



PMP Calibration

- Linear Calibration Program
- CES Parameter Calibration
- Exponential Cost Function Calibration
- Fully Calibrated Model

Step 1: Linear Program

- Regions: g
- Crops: i
- Inputs: j
- Water sources: w

Step II: CES Parameter Calibration

- Assume Constant Returns to Scale
- Assume the Elasticity of Substitution is known from previous studies or expert opinion.
  - In the absence of either, we find that 0.17 is a numerically stable estimate that allows for limited substitution

- CES Production Function

$$y_{gi} = \alpha_{gi}\left[\beta_{gi1}x_{gi1}^{\rho_i} + \beta_{gi2}x_{gi2}^{\rho_i} + \ldots + \beta_{gij}x_{gij}^{\rho_i}\right]^{1/\rho_i}$$

Step II: CES Parameter Calibration

- Consider a single crop and region to illustrate the sequential calibration procedure:

- Define: $\rho = \dfrac{\sigma - 1}{\sigma}$

- And we can define the corresponding farm profit maximization program:

$$\max_{x_j} \ \pi = \alpha \left[\sum_j \beta_j x_j^{\rho} \right]^{\upsilon/\rho} - \sum_j \omega_j x_j .$$

Step II: CES Parameter Calibration

- Constant Returns to Scale requires:

$$\sum_j \beta_j = 1.$$

- Taking the ratio of any two first order conditions for optimal input allocation, incorporating the CRS restriction, and some algebra yields our solution for any share parameter:

$$\beta_1 = \frac{1}{1 + \frac{x_1^{(-1/\sigma)}}{\omega_1}\left(\sum_l \frac{\omega_l}{x_l^{(-1/\sigma)}}\right)} \quad \textit{letting } l = \textit{all } j \neq 1$$

$$\beta_l = \frac{1}{1 + \frac{x_1^{(-1/\sigma)}}{\omega_1}\left(\sum_l \frac{\omega_l}{x_l^{(-1/\sigma)}}\right)} \frac{\omega_l}{\omega_1} \frac{x_1^{-1/\sigma}}{x_l^{-1/\sigma}}.$$

Step II: CES Parameter Calibration

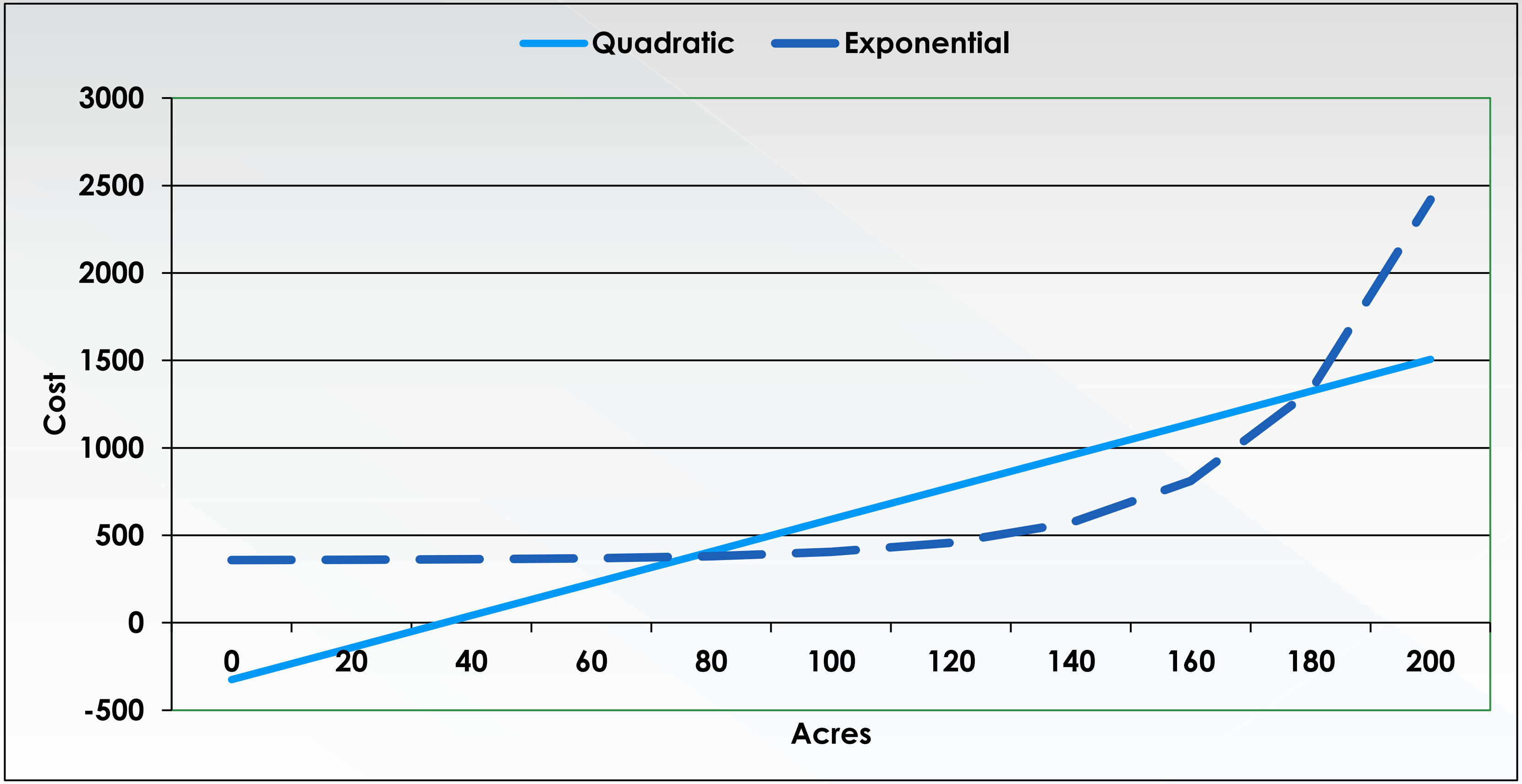
- As a final step we can calculate the scale parameter using the observed input levels as:

$$\alpha = \frac{(yld / \tilde{x}_{land}) \cdot \tilde{x}_{land}}{\left[\sum_j \beta_j x_j^{\rho}\right]^{\upsilon/\rho_i}}.$$

Step III: Exponential PMP Cost Function

- We now specify an exponential PMP Cost Function

$$TC(x_{land}) = \delta e^{\gamma x_{land}}$$

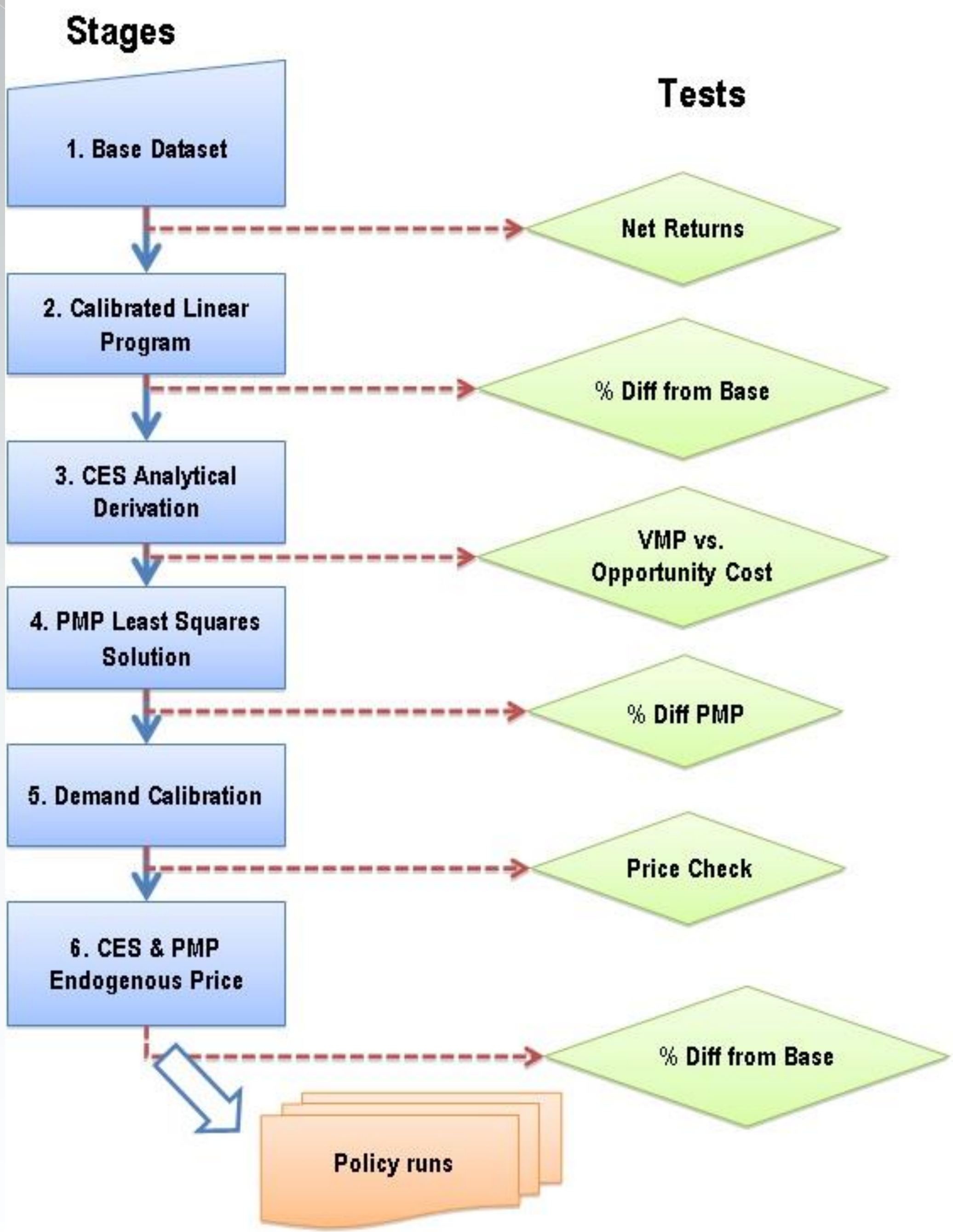


Step III: Exponential PMP Cost Function

- The PMP and elasticity equations must be satisfied at the calibrated (observed) level of land use
- The PMP condition holds with equality
- The elasticity condition is fit by least-squares
  - Implied elasticity estimates
  - New methods
    - Disaggregate regional elasticities

Step IV: Calibrated Program

- The base data, functions, and calibrated parameters are combined into a final program without calibration constraints
- The program can now be used for policy simulations


Stages
Tests
1. Base Dataset
Net Returns
2. Calibrated Linear Program
% Diff from Base
3. CES Analytical Derivation
VMP vs. Opportunity Cost
4. PMP Least Squares Solution
% Diff PMP
5. Demand Calibration
Price Check
6. CES & PMP Endogenous Price
% Diff from Base
Policy runs

Production Function Models: Extensions

- Theoretical Underpinnings of SWAP
  - Crop adjustments can be caused by three things:
    1. Amount of irrigated land in production can change with water availability and prices
    2. Changing the mix of crops produced so that the value produced by a unit of water is increased
    3. The intensive margin of substitution
  - Intensive vs. Extensive Margin